Lesson 7-3: Special Right Triangles

Think Man! Think!

There are two special right triangles determined by their angle measures. They are:

- 45-45-90
- 30-60-90

Your mission, should you decide to accept, is to determine for each triangle how the lengths of their legs relate with each other. I'll give you one *HUGE* hint…use the Pythagorean Theorem and assume a nice leg length:

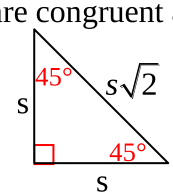
- 45-45-90:
    - Draw an isosceles right triangle.
    - What are the measures of the base angles?
    - Assume the equivalent length legs are x units long.
    - Determine the length of the 3rd leg in terms of x.
- 30-60-90:
    - Draw an equilateral triangle…what is the measure of each angle?
    - Assume a leg length of $2x$ units.
    - Bisect one angle forming two triangles out of the equilateral triangle.
    - What are the angle measures of these new triangles?
    - What is the length of the hypotenuse in terms of x?
    - What is the length of the base (bisected side) in terms of x?
    - Determine the length of the 3rd leg in terms of x.

Work through these before you go any further. These special triangles are ones you will see time and time again in your math career, especially next year in Trig. If you are like me, over time you'll forget what the leg measures for these triangles are. But, if you are like me, you will be able to remember how to build the triangles (above) and use the Pythagorean Theorem to reconstruct the leg relationships.

The 45-45-90 Triangle Theorem (Theorem 7-8)

In a 45-45-90 triangle, both legs are congruent and the hypotenuse is $\sqrt{2}$ times the length of a leg:

Hypotenuse = $\sqrt{2} \cdot$ leg

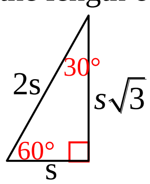


The 30-60-90 Triangle Theorem (Theorem 7-9)

In a 30-60-90 triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg:

Hypotenuse = $2 \cdot$ shorter leg

Longer leg = $\sqrt{3} \cdot$ shorter leg



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Examples

- For these types of problems, using mental math is much faster than using a calculator. Also the calculator answer will always be inexact (rounding), whereas squaring the square root of a number is always exact. Notice that for these problems, the answers are almost always left in root/radical form. The problem will tell you if you need to put the answer in decimal form with rounding.
- Also remember that we do not leave a root/radical in the denominator of a fraction.
- When working with a 30-60-90 triangle for which the length of the hypotenuse or longer leg is given, it is generally easier to start with finding the length of the shorter leg.

1. Find the length of the hypotenuse of a 45-45-90 triangle with legs of length $5\sqrt{6}$.

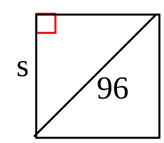
$$(5\sqrt{6})\cdot\sqrt{2}=5\cdot\sqrt{6}\cdot\sqrt{2}=5\sqrt{12}=5\sqrt{4\cdot 3}=5\sqrt{4}\sqrt{3}=5\cdot 2\cdot\sqrt{3}=10\sqrt{3}$$

2. Find the length of a leg of a 45-45-90 triangle with a hypotenuse of length 22.

$$s\sqrt{2}=22;\ s=\frac{22}{\sqrt{2}}=\frac{22\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}=\frac{22\sqrt{2}}{2}=11\sqrt{2}$$

3. The distance from one corner to the opposite corner of a square playground is 96 ft. To the nearest foot, how long is each side of the playground.

$$s\sqrt{2}=96;\ s=\frac{96}{\sqrt{2}}=\frac{96\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}=\frac{96\sqrt{2}}{2}=48\sqrt{2}\approx 67.88\approx 68\ ft$$



4. Find the lengths of the legs of a 30-60-90 triangle with hypotenuse of length $4\sqrt{3}$.

Shorter leg: $2\cdot s=4\sqrt{3};\ s=\dfrac{4\sqrt{3}}{2}=2\sqrt{3}$

Longer leg = $s\sqrt{3}=2\cdot\sqrt{3}\cdot\sqrt{3}=2\cdot 3=6$

5. The longer leg of a 30-60-90 triangle has length 18. Find the lengths of the shorter leg and the hypotenuse.

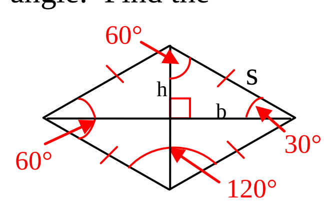
Shorter leg: $s\sqrt{3}=18;\ s=\dfrac{18}{\sqrt{3}}=\dfrac{18\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}=\dfrac{18\sqrt{3}}{3}=6\sqrt{3}$

Hypotenuse = $2s=2\cdot 6\sqrt{3}=12\sqrt{3}$

6. A garden shaped like a rhombus has a perimeter of 100 ft. and a 60° angle. Find the area of the garden to the nearest square foot.

$4s=100;\ s=25 \qquad 2h=s=25; h=12.5 \qquad b=h\sqrt{3}=12.5\sqrt{3}$

$$Area=4\cdot Area\Delta=4\cdot\frac{1}{2}b\cdot h=2\cdot 12.5\sqrt{3}\cdot 12.5\approx 541.27\approx 541\,ft$$



Homework Assignment

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